Electric Ducted Fan – theory and practice

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Table of Contents

Illustration Index

1 Introduction

The most common type of model aircraft seen flying under, radio control at model flying sites in the UK, is probably of the “spitfire” pattern – that is, *a model aircraft with propeller at the front driven by an electric or IC engine, a monoplane wing in the middle, and a rear tailplane and fin*. However, many modellers aspire to build and fly different types of aeroplanes (of which there are a very large number) such as biplanes and canards. Additionally, the motor and propeller can be mounted centrally, at the rear, on the wings and so on. In fact, model designs appear in a greater degree of profusion than is ever seen on full size aircraft in the modern era. This is in marked contrast to historical experience (in the period up to the second world war) when full size designs proliferated but models were rather simple and more uniform.

One type of model which has become very popular in recent decades is the “jet” style of aircraft, often conceived as *a scale, or semi-scale, model of a military jet fighter, or even a model of a commercial jet*. Tremendous strides were made with IC ducted fan powered model jet planes in the 1980's and 1990's. The complications of the design, and the marginal power of the motorised fans, resulted in these have been the province of a rather small set of very dedicated, and highly skilled, model aircraft enthusiasts, rather than the general model flying fraternity.

With the advent of electric powered model flight the possibility has arisen to drive ducted fans by electric motors, giving rise to Electric Ducted Fan models (EDF). More or less at the same time there has been a parallel development in model Jet turbines which develop considerably more power (albeit with a lot of extra complexity and cost). Consequently the jet model enthusiasts have more or less split into two camps:

- EDF powered small models, rather inexpensive, for general model enthusiasts
- turbine powered large models, rather expensive, for specialists.

In the following we shall consider only EDF as shown in the following examples:

Illustration 1: My West Wings Hawker Hunter.

This has a 32” wingspan with fixed undercarriage, powered by a Wemotec fan with Mega 16EDF motor and Overlander 2600mAh 40C Lipo battery. The model was built in January 2009 and first flown in May 2009.

Illustration 2: My West Wings Bae Hawk.

This has a 35” wingspan with electric retracting wheels, powered by a Wemotec fan with Mega 16EDF motor and Overlander 3200mAh 60C Lipo battery. The model was built in February 2011 and first flown in June 2011.

Illustration 3: The Max-Thrust GR4 Tornado

This has working swing wings, powered by a 50mm ducted fan unit. This model is all foam, and has a low wing loading, hence its ability to take off from the ground (rather than bungee launch).

Illustration 4: The E-flite Habu

This is powered by an 80mm 5-bladed fan. This model has retracts wheels, flaps all normal functions. Its performance is such that it is considered a suitable model to train on for those interested in later moving to turbine powered planes.

1.1 Objectives of EDF

EDF power units are defined as ***an electric motor driving a small, multi-bladed, impeller housed within a cylindrical shroud***.

Illustration 5: A wemotec 70mm 5-bladed fan with Mega-EDF motor fitted

The advantageous properties of an EDF unit are:

- The ducted fan unit is ***compact and enclosed*** and can therefore be mounted inside a fuselage, as well as externally. This makes it a suitable for powering a jet type of aircraft.
- Being electric the unit has no need for fuel tanks, throttle servo, exhaust ducting, starting equipment, ignition and do on, and is therefore ***easier to construct*** and ***easier to operate*** than IC powered fans.
- The shrouded propeller should be ***more efficient*** that a propeller of equal size in free air, as the losses due to tip vortices, pressure decrease at the blade tip, and so on are reduced.
- The potential exists for ***vectored thrust*** by diverting the exhaust flow direction to a small extent, which may be useful for some aircraft to perform certain unusual manoeuvres.
- Multiple engined models are much easier to build than an equivalent IC engined plane.

The disadvantageous properties of an EDF unit are:

- The ducted fan unit produces less thrust and is therefore ***less effective*** than a normal propeller driven by an equal amount of power. It is generally of smaller diameter, and has a higher propeller blade loading, than the two-bladed propeller for the same model.

- In order to increase the thrust of a given EDF unit it is usual to increase the power and speed of the motor, but this requires ***more Watts of energy***, which in turn requires ***a bigger battery*** capable of higher current draw rates.

An example of the sort of units suitable for a multi engined plane, such as a learjet, is shown in the net picture.

Illustration 6: A pair of 60mm EDF 3-blade units

These fans are suitable for external attachment to wings or fuselage

1.2 What do we need to know ?

The sort of questions that arise when planning an EDF-powered model are:

- How to choose the best EDF unit for a given model
- How to design a new model around a particular EDF unit
- How to check the performance of the model and EDF unit is as it should be
- How to improve the performance of a given model set up.

What is needed is an understanding of how fans work (i.e. the physics of the ducted fan propulsion process) and therefore a method of solving problems, making adjustments and so on.

1.3 What don't we need to know ?

A considerable amount of "mis-information" or "dis-information" seems to exist about the mechanics of ducted fans. Some of these misconceptions will be addressed here (and the reasons why they are wrong).

Some common ideas (which are NOT true) are:

- *"a ducted fan speeds up when the model gets in the air, so that the model flies faster"* (not true). It is often claimed that the "note" emitted (the whine of the motor) increases in pitch because the fan moves into a more efficient regime ("gets on the step") and therefore produces more power. The note may in fact, rise, although this is hard to prove, and is confused by the Doppler effect when the plane is approaching the pilot (it seems to be accelerating, and the apparent pitch of the sound does increase). But there is no regime change in EDF operation: in fact the power produced by a fan unit in flight actually decreases with increasing speed – if it didn't the plane would eventually reach the sound barrier).

- *"ducted fan planes need to be bungee launched to get them flying fast quickly"* (not true). It is often observed that bungee-launched models seem to "sag" as they leave the bungee, but quickly pick up speed. This is due to the fact that most EDF planes under underpowered for their weight and thus need time to accelerate to flying speed. The bungee gives good initial acceleration, but the low power unit is slow to take over. The same applies to hand launching. As EDF plane designs and equipment has improved planes are able to take off from a ground run as is normal for other model aircraft.

- *"a ducted fan is an electric jet and produces power in the same way a gas turbine does"* (not true). An EDF unit is similar in principle to the compressor stage of a gas turbine, but only that. An EDF unit does not heat air, combust fuel or produce large quantities of surplus thrust. For the sake of simplicity we call them "electric jets" but this is a misnomer.

- *"a ducted fan plane can be made to go better(faster) by substituting a more powerful motor"* (not always true). An EDF unit depends on applying some Watts of electrical energy to push air backwards, and therefore push the model forwards. More watts applied should make for more air pushed faster. However, a higher wattage motor may be heavier, and will require a bigger battery, which will also be heavier. The airframe may need to be strengthened to withstand that increased strains of the motor and the increased aerodynamic forces on the flying surfaces, resulting in a heavier airframe. The higher overall weight of the plane will result in a higher wing loading making the plane difficult to take off, liable to stall in flight, and to have a higher landing speed – all undesirable characteristics. Longer flight times, more manoeuvrable flight, and better flying characteristics can often be better achieved by decreasing the all up flying weight of the plane.

1.4 A digression on Jet Engines (Gas Turbines)

Gas turbine engines (jet engines) work by

1. ***using a compressor*** to convert incoming air to high pressure, then
2. ***combusting fuel*** to produce a hot jet exhaust, and finally
3. ***using a turbine*** to extract some of the exhaust energy to drive the compressor.

The expansion of the burning fuel creates enormous energy in the form a high pressure exhaust stream which can be used in various ways:

- as direct thrust, similar to rocket propulsion in a supersonic ***jet*** (as used on many military fighter aircraft)
- to make the turbine power a propeller – the ***turbo-prop*** engine (as used on many small and intermediate commercial planes)
- to make the turbine power a fan – the ***turbo-fan*** engine (as used on most larger commercial airliners).

An EDF unit is similar in principle only to the compressor stage of a jet engine. It is a very inefficient compressor relative to the axial compressor of a gas turbine engine, or even the centrifugal compressor of a non-axial jet engine. In all other respect an EDF unit is totally different to a gas turbine as it does not greatly compress incoming air or combust fuel. As a result it cannot produce large quantities of surplus thrust.

The corollary is that, since an EDF unit is a low-power thrust device, it will only have application in certain areas, such as:

1. small planes carrying small batteries, but with potentially powerful motors
2. very light planes carrying small batteries
3. multi-engined planes, where motors can potentially share batteries

It is notable that to date (2012) the most successful EDF model planes use military fighter scale appearances, are of foam construction, are of the order of 1 metre wingspan and weigh about 1.5Kg. Below these limits most of the weight of the plane is equipment (fans, motors, batteries, radios, etc.) and the airframe tends to be rather fragile. Above these limits and the total all up weight is rather high, requiring more thrust to generate sufficient lift.

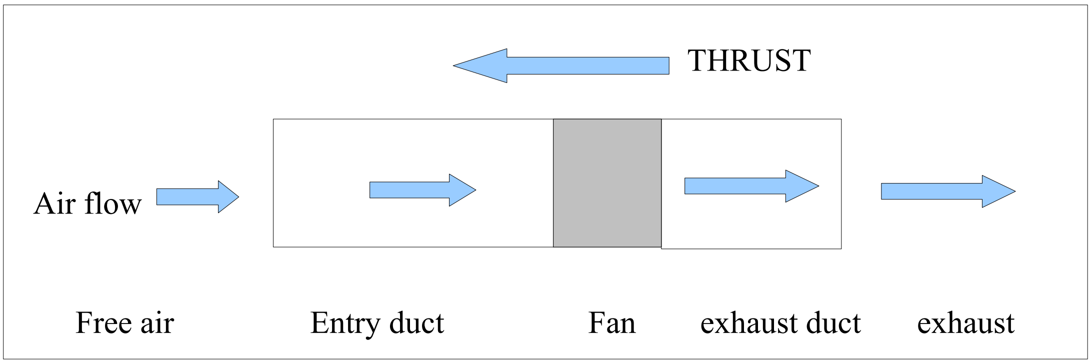
1.5 The aim of this document

Surprisingly there is little scientific literature written on the theory of how EDF units work. There is, of course, a great deal of un-scientific literature on the subject, most of it wrong, misleading or incomplete. Information on how to make better units, and make model planes fly better is also scarce. The most important omission is a readily available means to calculate the thrust produced by a fan unit for help in selecting the right unit to use, matching it to a speed controller (ESC) and battery appropriate for the type of aircraft and flying style desired. This document seeks to fill some of these gaps.

Some articles which appeared in various model magazines, starting with an MFI magazine article of 1984 in which Klaus Scharnhorst derived an equation for the velocity of air exiting from a static fan (i.e. one strapped to the bench). This value can then be checked against observation. This work culminated in a theory of EDF operation published in 2005[1] (in English). The velocities of air into and out of, an EDF system are related to the power required to drive the fan in both static and dynamic cases. Given the geometry of the model (ducts, fans, etc.) and other information (such as the speed of the model) we can work out how much power is needed, and therefore what motor, battery is required.

2 A simple explanation of EDF propulsion for model aircraft

The components of an electric ducted fan are exemplified in the following diagram:



A simple depiction of an EDF system in which air is allowed to flow in (at the left) and is pushed out (at the right), resulting in a force on the airframe (to the left).

2.1 The components of an EDF system

A Ducted Fan system can be thought of as the following components, arranged in order:

- an ***entry duct***, or ***forward duct***, which channels air from an entry aperture to the front of a fan unit. In some cases there may be two entry apertures, one on each side of the fuselage. These usually join at a confluence ahead of the fan unit. The forward duct is not usually cylindrical (although it could be). More commonly the forward duct is tailored to the shape of the fuselage it is mounted in.
- A ***fan unit*** which comprises a short cylindrical tube, in which is mounted a motor driving a fan (or impeller). The impeller may have 2, 3, 4, 5, 6 or more blades according to its design. The entry and exit diameter of the fan unit is almost always the same. Note that the presence of the motor in the centre of the fan unit automatically takes a small amount of the fan area away. The remainder of the space

through which air flows is called the ***fan-swept-area*** (FSA).

- An ***exhaust duct***, or ***rear duct*** which connects the fan unit to the outside. The rear duct, or exhaust, is most often cylindrical or nearly so. The rear duct usually tapers to the exit by a small amount. A little tapering causes a beneficial speeding up of the exhaust flow and generates more thrust. Too much tapering introduces resistance into the system which causes a lack of thrust.

2.2 The operation of an EDF system

The following section attempts to give a simple, non-technical description of how and why a ducted fan unit produces sufficient propulsion to power a model aircraft.

Evidently, the EDF unit is connected to a battery, via an ESC (electronic speed controller) under the command of a radio receiver supporting a throttle control servo channel. The pilot sets the throttle to a given level, the fan produces an airflow through the ducts resulting in a reactive force on the airframe and the plane is pushed forward.

The various components of this picture are described in a little more detail:

1. In front of the entry duct (the area into which the plane is flying) is a free air-stream This air may be moving (according to the wind at the time) and may be flowing towards the plane (the plane is flying upwind), or away from the plane (the plane is flying downwind), or in some cross-wind direction. To simplify the discussion we assume here that there is no external wind and the plane is flying in still air. This is unrealistic as in natural conditions there is always some wind, but it will do for a theoretical abstraction, since in practice the wind velocity as low relative to the model velocity (the plane is flying fast). Remember, too, that a plane flying in a moving air-stream is also moving with the air-stream Relative to the pilot on the ground it may appear to fly slowly in one direction and fast in another, but of course relative to the air it is flying at the same speed most of the time.
2. In the entry duct air is moving along the duct, essentially at the same speed that the plane is flying. There will be some resistance to flow caused by friction on the walls of the duct, but this can be considered to be small (air is not very viscous, and the duct is not so small that the resistance would be significant).
3. As the air passes through the fan unit it is slightly compressed, and therefore its temperature will in fact be raised by a small amount (probably too small to measure). This is the result of the rotating fan blades compressing the air. Considerable energy is exerted by the motor to turn the fan at a relatively high speed. In a typical example 350 watts of electrical energy (11.4 volts at 30 amps) is used to turn a fan at 45,000 rpm generating around 9 Newtons of force.
4. In the exhaust duct the slightly compressed air escapes to the outside atmosphere, and reverts to normal atmospheric pressure. The escaping air is in the form of a stream of faster moving air. This creates an equal and opposite force on the airframe which pushes the plane forward.

Some observations can be made immediately: Air must pass easily through the ducts and fan without restrictions, otherwise the EDF unit will not operate at its best. We know already that it is not likely to have power to spare, so none must be wasted by tortuous or long ducts, narrow openings, and so on.

3 A theory of Ducted Fan propulsion

This treatment follows the presentation of Klaus Scharnhorst. It is suggested where it might be incomplete. A more accurate treatment is proposed later.

3.1 Assumptions about the physics of airflow and motion

The following assumptions are made:

- the airflow is ***incompressible***.
- The effective size of the EDF unit is determined by the ***Fan Swept Area.***
- The ducts are ***frictionless***.
- Ducts can be assumed to be ***circular in cross section***.
- The ***bifurcation*** and joining of ducts can be ignored.

Actually, air as a fluid is ***compressible***, as will quickly be noticed when you pump up a bicycle tyre, for instance. The pump increases the pressure of some air which is then stored inside the tyre. However, heat is also produced by this process (the pump warms up). Similarly, when air is compressed in the first stage of a gas turbine, it heats up, to approximately the temperature at which the second stage combustion process works.

Temperature rise is not significant in the case of the flow of air through an EDF unit.

The increase in air pressure caused by the fan does not cause the exhaust duct to bulge out because of high pressure, nor does the entry duct collapse because of low pressure. Actually this can happen if the ducts are not properly built, but it is rare. The unit does not get hot, or freeze (although the electric motor may generate heat because of electrical inefficiency).

FSA is defined as the area of the impeller minus the area masked by the motor. If the diameter of the EDF unit is d_u and the diameter of the motor is d_m, then the FSA is defined as:

$$\text{FSA} = \pi (d_u/2)^2 - \pi (d_m/2)^2 = \pi (d_u^2 - d_m^2) / 4$$

3.2 Defining the properties of an EDF system

One of the problems in calculating useful properties of EDF units, and the resulting performance of the plane for which this supplies power, is having some measured values to start from. It is no use having a method of calculating the exhaust velocity from the mass flow induced by the fan if we have no way of measuring that in the first place.

3.2.1 The geometry of a ducted fan system

The geometrical properties we can measure on the plane are:

the entry duct diameter is d_l, and therefore the entry duct area is $A_l = \pi (d_l/2)^2$. Note that in the case of an entry duct which is not circular, which may be common for many scale-appearance planes, the area can be measured in other ways, and if needed, the effective diameter calculated.

The entry duct length is l_l. This may be important if duct losses are expected to be significant,

The fan diameter is d_f, and therefore the fan area is $A_f = \pi (d_f/2)^2$

The motor diameter is d_m and the motor cross-sectional area is A_m.. This is often given by the manufacturer. Remember to include any thickness added by the motor mounting cylinder,

The fan-swept-area is $A_{sw} = A_f - A_m$.

The exit duct length is l_2.

The exit duct final diameter is d_2 , and therefore the exit duct area is $A_2 = \pi (d_2/2)^2$

3.2.2 The electrical properties of a ducted fan system

The electrical properties we can measure on the bench in static testing are:

the voltage, v, at which the motor operates. e.g. v = 11.4 for a 3s Lipo battery.

the amps, a, drawn by the motor at various throttle settings, including the maximum throttle position. e.g. 35 amps

The watts, w, being consumed by the motor and fan running at maximum speed. e.g. power consumed equals 11.4 volts * 35 amps = 399 watts. Note that this is not the same as the power produced (thrust generated) by the EDF unit because of inefficiencies in the conversion of electrical power into useful work. We shall return to the question of the efficiency of the EDF system later.

3.2.3 The aerodynamic properties of a ducted fan system

The aerodynamic properties we can usefully measure in the air, when the model is in flight, are:

the airspeed of the plane, u, in metres per sec, m/sec. This can be measured by a pitot tube airspeed indicator (as on full size planes) and recorded by an onboard logger.

The voltage, amperage and wattage of the system if we have an on-board logging system with appropriate sensors.

The airflow through and over a ducted fan system is shown in the following diagram. The streamlines represent lines of equal pressure and denote the path taken by a individual air particle.

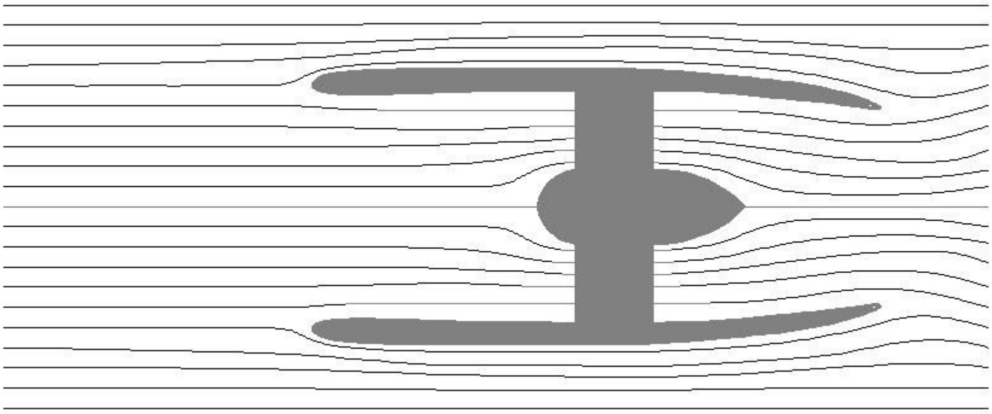


Illustration 7: Normal airstream flow through a Fan unit at flying speed

3.3 Calculating the Thrust of an EDF system

This section is a brief summary of the Klaus Scharnhorst paper. The EDF system is considered (for the moment) to be a single "back box" system. Looked at from the outside it has an entry and an exit, through which air passes, and which producs thrust. We don't need to consider the internals in too much detail at the moment.

To ensure that the values we are calculating are consistent and meningful, we need to keep track of the dimensions of the various quantities using dimensional analysis. In this analysis the following terms are used:

- M for a quantity of mass. The unit of mass is the kilogram,

- L for a quantity of length. The unit of length is the metre,
- T for a quantity of time. The uit of tim is the second.

Thus a quantity of area is L^2, and quantity of volume is L^3. Velocity has dimensions LT-1. Mass flow has dimensions MT^{-1}, and force MLT^{-1} which we call Newtons. Power has dimensions of ML^2T^{-2} which we call watts.

3.3.1 Conservation of mass

When the plane is in straight and level flight it is not accelerating or decelerating,:

The inlet area is A_i and air enters this with velocity v_i (the speed of the model). The volume of air passing through this in unit time is

$$Q_i = A_i v_i.$$

With dimension L^3T^{-1}. Note that area is measured in square metres, or m^2, and velocity in m/sec.

The air entering must exit somewhere. The volume of air exiting in unit time is

$$Q_e = A_e v_e$$

where A_e is the exit area and v_e is the exit velocity. Since there is no combustion, or temperature change involved, we can assume that air is incompressible. Now, as a result of the principle of continuity for fluid flow:

$$Q_e = Q_i.$$

Therefore, re-arranging this equation, we can arrive at an equation for the velocity of the exhaust, which is:

$$v_e = (Ai / A_e) * v_i$$

3.3.2 Mass and thrust

The mass of air being moved is

$$M = \rho Q_i$$

where ρ is the density of air (= 1.224 kg/m^3 at Standard Sea Level). This has dimensions of MT^{-1}. Mass flow is measured in kg/sec.

The speed up of the air produced by the fan is the difference between the speed of the incoming air and the speed of the exiting air, or

$$dv = v_e - v_i .$$

The thrust is the force produced by the extra moving air and is given by the momentum increase of the moving air mass, which is the mass being moved multiplied by the increase in speed, or

$$T = M\ dv.$$

Thrust is a force which has dimensions of MLT^{-1} and is measured in Newtons. One Newton is the force of an apple falling under gravity, or about 3.5 ounces = 102 grams.

3.3.3 The power needed for flight

We can denote the power needed for flight as P_{flight} which is a force applied at a given speed. It therefore has the dimensions of $MLT^{-1} * LT^{-1}$ or in total ML^2T^{-2} which is called watts. The power needed for flight, measured in watts, at this speed is:

$$P_{flight} = T\ v_i$$

3.3.4 Balancing the forces acting on the EDF system

Another way of looking at the EDF system is to calculate the force produced by the fan in terms of all the forces acting on the system. By the principle of conservation of energy (energy may be neither created nor destroyed) the forces must sum to zero when all are considered.

The power gained by the system because of the input of flowing air is P_{gain}, is given by the kinetic energy of the mass of air entering the system. The kinetic energy of an object (here, the moving mass of air) is given by Newton's laws of motion as a mass for a distance, times an acceleration. It therefore has dimensions of $M * L * LT^{-2} = ML^2T^{-2}$, which is in watts . It is defined as:

$$P_{gain} = \tfrac{1}{2} M\ v_i^2$$

The power lost from the system, P_{loss}, caused by ejecting a mass of flowing air is, similarly, given by the kinetic energy of the exhaust, and is:

$$P_{loss} = \frac{1}{2} M v_e^2$$

So, using the conservation principle, we have

$$P_{fan} - P_{loss} + P_{gain} = 0$$

The power produced by the system, P_{fan}, which is due to the fan working away inside is again measured in watts, and is, therefore:

$$P_{fan} = P_{loss} - P_{gain}$$

3.3.5 Finding the efficiency of the EDF system

The "efficiency" of the fan is the nearness to which the energy required is matched by the energy provided, or

$$\text{Efficiency} = P_{flight} / P_{fan}.$$

As a percentage this is usually around 80%.

The Power required from the motor, P_{motor}, also depends on the losses in the system due to friction in the duct, restrictions in the geometry of the duct and so on. KS assumes (from an analysis of pipe flow theory, etc..) that $P_{motot} = P_{fan}/0.85$ is a reasonable approximation. The efficiency of the motor is therefore:

$$\text{efficiency(motor)} = P_{fan} / P_{motor} = 0.85 \text{ approx.}$$

3.4 A summary of what has been achieved

The theory outlined above is remarkable in that it has created a logically reasoned argument which connects the static properties of the EDF system to the power budget of the system in operation. This is by no means a small achievement.

In summary we can describe this theory as a method which requires ONLY the size of the entry and exit ducts, AND the speed of the model in level flight, to determine the power required for flight. From a consideration of other physical properties we can also find the efficiency of the motor.

In terms of the flying performance of the EDF powered plane we can say that:

> ***IF the plane is of the given physical specification, AND if flies at the given speed, THEN the thrust required for flight is the one calculated.***

To use this theory to calculate a useful design property such as the power required from the motor (ie which motor to choose to install):

> ***Calculate the power required for flight from the geometry of the ducts and the assumed flight speed. Then calculate the required power of the fan,and finally the required power of the motor.***

Note that these considerations only apply to the plane in straight and level flight at its cruising speed. If the plane is to take off from the ground and climb to a reasonable flying height, and if it is expected to do some interesting aerobatics, then of course more power will be required. So,

> ***The power calculated is the minimum required for flight.***

3.5 An Example – the West Wings Bae Hawk in level flight

This example uses observed values from my West Wings BAE Hawk in actual flight.

The area of the inlet is 3667 mm^2 (= 0.003667 m^2) and the area of the outlet is 2463 mm^2 (= 0.003667 m^2)

But what is the speed of flight ? It is *much* faster than my scale Tucano (which flies around 60mph as measured by on-board EagleTree telemetry) , but not as fast as a pylon racer (known by timing trials to be of the order of 180mph). This creates a difficulty (which we could resolve by fitting an [Eagle Tree] logger measuring airspeed). For the moment let's assume it could be 37m/sec = 81 mph as a reasonable guess.

Then, the volume of air being consumed at the inlet, per second, is Q = 0.14 m^3 with a corresponding mass flow of M = 0.17 kg/sec.

So, calculating the speed of the exhaust we obtain v_e = 55.1m/sec (about 110 mph)

The force acting on the plane is the thrust T = 3.0 Newtons (=10.8 oz = .307 grams).

The power needed for flight is then P_{flight} = 111.2 watts.

From the conservation of energy, the power of the fan is P_{fan} = 138.4 watts, assuming a fan system efficiency of 80% .

Assuming a motor efficiency of 85% we estimate that the required power of the motor is P_{motor} = 163 watts.

The actual power consumption in flight, as measured by total battery depletion over flight time, is obtained as follows: a 3s 2200mAh battery was exhausted in a 5 minute flight consisting of a take off run at full throttle, some level flight at two thirds throttle, as a landing approach and touchdown at one third throttle. So,

> 2200mAh = 2.2Ah = 132 Amp minutes.
> The average amperage over the flight is therefore 132 / 5 amps = 26.4 amps.
> A 3s battery at 11.4 volts, so average consumption is 26.4 *11.4 = 301 watts.

The efficiency of the motor therefore seems to be 45% rather than 85%, why is this ? Clearly, some loss of power goes to making the ESC hot, the motor hot, the battery hot, etc. There is some measure of agreement between theory and practice, but perhaps the difference is due to over use of maximum throttle in the take-off and climb phase of flight before the straight and level test began. Consequently, the duration based calculation of power required is an over-estimate. However, every flight requires a take-off phase, so the 45% average efficiency may be a more realistic practical guide.

3.6 The theory of Static thrust

Note that the figures given in the last example are NOT the figures for static thrust and power consumption as obtained on the bench. And yet the static thrust on the bench is the only thing we can measure prior to the maiden flight. If we want to be sure that the plane will fly on its maiden flight we need to know the relationship between the static thrust measurement and the dynamic thrust expected in flight.

The whole problem of understanding the STATIC thrust is also analysed by KS.

The important point is that we can EASILY measure static thrust with a dynamometer on the bench, and we can easily measure static power consumption with an ammeter on the bench, but we cannot easily measure either of these properties in the dynamic flight environment. Yet dynamic thrust is what we want to know, since that determines model performance. What we need is to calculate dynamic thrust from static thrust.

3.6.1 Assumptions and methodology

The EDF system is assumed to be held in a static position for testing. For example the complete plane, ducts and fan, can be anchored to a workbench. When the system is powered up and the fan made to pump air out of the exhaust duct the airflow through and around it will be as shown in the following diagram:

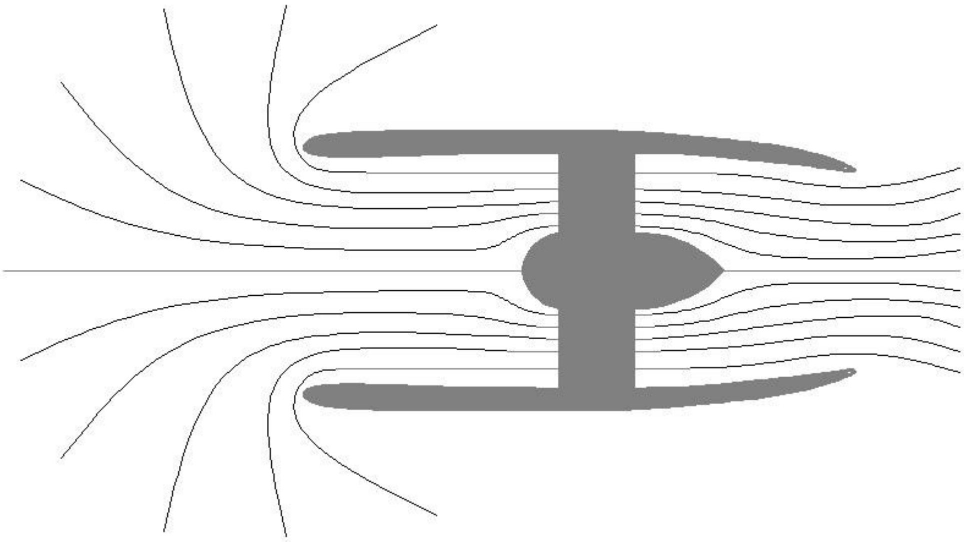


Illustration 8: Airstream through an EDF unit when it is not moving (static)

Note, especially, that while the exhaust airflow looks much as it did in the dynamic case, the inlet flow looks very different. Air is being drawn from a large volume around the inlet opening, which implies that the effective inlet area is much larger, and the velocity of the incoming air is correspondingly much slower, than in the dynamic case.

The methodology of this section is to work backwards through the equations we developed above in order to find the value of v_e from the observation of the power consumed in the bench test. Once we have an estimate of the exhaust velocity we can calculate the thrust in the same way as before.

3.6.2 Assumptions made for the static analysis

Of course the continuity requirement means that an amount of air must be drawn in at the front and the same quantity expelled at the back. However, the difference between the inlet velocity and the exhaust velocity is now much more extreme. We will therefore make the approximation that $dv \rightarrow v_e$ (read this as “dv tends to the value of v_e) and in the limit we can assume that $dv = v_e$.

Also, we will assume that since v_i is very low, the kinetic energy gained by the system as result of the incoming air, P_{gain}, is very low. In fact, since the air entering the system is doing so as a result of work done by the fan (sucking air in from a region of still air around) that it is reasonable to assume that $P_{gain} = 0$.

Finally, we will assume that the electrical power consumed by the motor is the same as that consumed at the same throttle setting in flight. But note that this is a quantity which we can also measure on the bench since we can connect an Ammeter / Volt meter in between the battery and the ESC.

3.6.3 Calculating the exhaust velocity from the system power

From the energy balance equation we can substitute $P_{gain} = 0$ and obtain

$$P_{fan} = P_{loss} - P_{gain} = P_{loss}$$

In the assumptions above we decided that it was reasonable to make P_{fan}(dynamic) = P_{fan}(static) so that now we know the value of P_{loss}(static).

From the kinetic energy definition we know that

$$\begin{aligned} P_{loss} &= \tfrac{1}{2}\ M\ v_e^2 && \text{from the kinetic energy equation} \\ &= \tfrac{1}{2}\ Q_e\ \rho\ v_e^2 && \text{from } M = Q_e\ \rho \\ &= \tfrac{1}{2}\ A_e\, v_e\ \rho\ v_e^2 && \text{from } Qe = A_e\, v_e \\ &= \tfrac{1}{2}\ A_e\ \rho\ v_e^3 && \text{by collecting terms} \end{aligned}$$

Therefore, by re-arranging this equation, and solving for v_e we obtain:

$$\begin{aligned} v_e^3 &= (2\ P_{loss}) / (\ A_e\ \rho\ ) && \text{by re-arranging, and then} \\ v_e &= \{\ (2\ P_{loss}) / (\ A_e\ \rho\ )\ \}^{-3} && \text{by taking the cube root of the expression.} \end{aligned}$$

This equation for the value of the exhaust velocity in the static case was first developed by Klaus Scharnhorst in the 1980's and published in various magazine articles. It has probably not got the attention it deserves as a truly remarkable step forward, maybe due to the length of development which necessarily proceeds it, and which has been reproduced here.

3.6.4 Calculating Static Thrust

Now that we have an expression for the exhaust velocity in the static case it is simple to find the thrust produced in the static case, since $T = M * dv$ as before, and we have made the assumption that $dv = v_e$ at least approximately. So we now have:

$$T(\text{static}) = M\ v_e$$

3.7 An example – the BAe Hawk in static test

The Bae Hawk was anchored to the bench and the motor run to perform the static test. In this configuration the battery is connected to a meter which us then connected to the ESC. The meter displays instantaneous volts, amps and watts on the main battery circuit.

3.7.1 Using the static case theory

First calculate the expected (dynamic) thrust from the results of the dynamic case as calculated above. We accept that the motor should use the same power in the static case as in the dynamic case, that is 163 watts. It has the same efficiency as before, 85%, and so the power required by the system is $P_{sys} = 138$ watts.

We have assumed that $P_{gain} = 0$, and so $P_{loss} = P_{sys} - P_{gain} = 138$ watts.

Using the equation developed above we can make the following steps:

$$v_e = \{ (2 \ P_{loss}) / (A_e \ \rho) \}^{-3} = 45.1 \ \text{m/sec}$$

The first thing to note is that the (static) exit velocity calculated is ***lower*** in this case than in the dynamic case – why is this ? Remember that in the dynamic case air is entering the EDF at flying speed and the EDF is accelerating the air a bit more, whereas in the static case air is entering the EDF at a very low speed. This slow air is then being accelerated greatly, but in total not to the same speed as is achieved in the dynamic case.

We can also calculate the mass flow, M(static), and the static thrust, T(static) using the standard equations:

$$M(\text{static}) = A_e \, v_e \, \rho = 0.14 \ \text{k/sec}$$

$$T(\text{static}) = M \, dv = 6.139 \ \text{Newtons} = 22 \ \text{oz} = 626 \ \text{grams}$$

Notice that the mass flow is slightly less in the static case than in the dynamic case (as a result of the lower exit velocity) but the surprise is that the static thrust is greater than the dynamic thrust. In fact it is double. What this seems to be saying is that ***the EDF unit that powers the plane to fly straight and level at the specified speed with a thrust of about 3 Newtons should produce a static thrust of over 6 Newtons on the bench***, if we did that test.

An alternative way of expressing it would be to say that ***if this EDF system is observed to produce 6 Newtons of thrust on the bench then, when flying at the given speed, it will be producing only 3 Newtons of thrust in the air.*** It is another set of considerations now to decide whether the 3 Newtons is sufficient to fly the aircraft.

3.7.2 Using the observed current consumption

Since we can run an EDF system on the bench we can easily measure the current it actually consumes, and therefore gain an insight into whether some of our assumptions are correct. In the case of the Bae Hawk, runnning it up to maximum throttle gives a set of values for the current used as follows:

Battery type	Volts	Amps	Watts
3S 2200mAh 40C	10.3	27	283
3S 2600mAh 40C	10.3	30	317
3S 4000mAh 30C	10.2	32	353
4S 2200mAh 60C	13.2	39	514
4S 3200mAh 25C	12.8	38	488

These results clearly show that the bigger the battery capacity, and the higher the battery voltage, the more watts are consumed (and therefore hopefully the more thrust is produced). Additionally the C-rating of the battery has an influence on how much current the battery can deliver in a given time.

The target battery for the plane is the 3S 2600mAh size of battery. Larger batteries carry an additional weaight penalty, so the choice of battery is a compromise between the weight and the thrust produced.

If we take the 317 watts as a typical bench mark it would seem that the efficiency of the EDF unit (motor, fan and duct system) is really 138 / 317 = 44%, not the 85% previously assumed. There are a couple of caveats around this conclusion, however:

1. the bench test was full throttle, whereas the plane flying straight and level will (hopefully) not be at full throttle. Until we fly it we don't actually know what throttle level it will require. It will probably be about 2/3rds throttle, and the power consumed with therefore be more like 200 – 250 watts, indicating an efficiency of about 60%.
2. the motor and fan unit have unknown inefficiencies. While the (brushless) motor may be 90% efficient the fan blades themselves may be inefficient in turning this into thrust, there may be leakages around the edges of the blades, and so on.
3. The ducts are of unknown efficiency. Clearly the rather long, and curved intake ducts are responsible for some losses (over the straight nacelle shape shown in the diagrams. The question of duct efficiency needs to be dealt with separately.

3.7.3 Using the observed thrust measurement

Using the selected battery (at the selected current consumption) the static thrust can be measured by placing the plane on a weight balance (digital kitchen scales are appropriate for this test). Note that the plane should be mounted vertically on the scales, the scale's zero reset, and then power applied. The thrust produced is then observed as additional weight which is read off from the display panel of the scales.

In the case of the Bae Hawk the measured (static) thrust was 440 grams = 4.313 Newtons = 15.3 oz at 317 watts for full throttle.

This value is a little less than the calculated value of 6.139 Newtons which suggests that the theory is only approximately correct. No doubt some additional considerations need to be taken into account (and will be discussed below).

Now working backwards through the methodology we can see that the assumption about the flying speed may have been wrong. We have (as yet) no actual observation of this value which was assumed (guessed) in the first instance. If the flying speed had instead been estimated at 31 m/sec = 68 mph then the static thrust calculated would be 4.309 Newtons, and the corresponding dynamic thrust would have been 2.11 Newtons.

In order to verify this we need to be able to do one of two things: either:

- calculate (by airfoil lift/drag considerations) whether enough lift would be produced at the lower flying speed, or
- measure the actual speed in flight.

3.7.4 A possible adustment to the KS theory

One of the well-voiced objections to the KS theory is that it seems to imply that in the static case the entry velocity of incoming air is zero, or effectively so. Clearly this cannot be literally true, although we know the value of v_i is small.

In concept the EDF unit in the static case is operating a little like a vacuum cleaner, sucking air in at the front and expelling it at the back. The airstream diagram above shows that air is drawn from all around the EDF inlet, and the rounded lips of the intake duct are clearly essential in guiding the incoming in the right way. The result of this is that the effective intake area is much larger than the actual intake size (which is the controlling dimension in the dynamic case). Vacuum cleaners make use of this fact by reducing the inlet nozzle size to a value as small as is practicable.

If we make the reasonable assumption that the effective diameter of the intake is about double the actual size in the static case, we can then make an estimate of the average intake velocity, since we have

$$A_i * v_i = A_e * v_e$$

and so by re-arranging,

$$v_i = (A_e / A_i) * v_e$$

The new value of the inlet area can be calculated from the area of a circle with a diameter of twice the original.

In the case of the Bae Hawk the inlet diameter is now 2 * 68.3 mm = 136.7 mm and the corresponding inlet area is 14668 sq mm = 0.014668 sq m. Using the formula above for the case of flight speed = 37 m/sec and dynamic thrust = 3.007 Newtons we obtain the exit velocity of 45.1 m/sec as before. But now if we take account of $dv = v_e – v_i$ we obtain a value of vi = 7.6 m/sec and therefore dv = 45.1 – 7.6 = 37.5 m/sec. The revised value of static thrust is now T(static) = M * dv = 0.14 * 37.5 = 5.108 Newtons.

As before if we adjust the estimate of the flying speed until we get agreement we arrive at a figure of flight speed = 34 m/sec giving a T(static) of 4.313 Newtons corresponding to a dynamic thrust of 2.539 Newtons.

3.8 Conclusions on the utility of the static test method

The following conclusions on the utility of the static analysis can be made:

1. The dynamic calculations, and the associated static calculation, can be conisdered a generally good guide to basics of EDF operation. We have a logical understanding of the mechanism of the EDF unit and its context in an actual plane. We have a systematic was of estimating the power needed to make a plane fly at a given speed, and we have a way of relating the measurement on the bench (the static case) to what might happen in flight.
2. On the other hand, the calculated figures are of the right order of magnitude but some significant discrepancies with respect to observations. Some of this is due to the difficulty of making accurate measurements on a real plane, and some of it is due to assumptions that have been made during the calculations. Clearly both of these factors need to be refined.

3. The surprising result that ***static thrust* > *dynamic thrust*** means that simply taking the result of a bench test and using it to decide on motor selection, battery selection and so on may be misleading. What has been missing all along is an aerodynamic calculation of how much lift is required to fly the plane in the first place, since

   lift = weight in straight and level flight.

   The lift generated by the airfoil will depend on the profile of the airfoil (its coefficient of lift) and the size of the airfoil (its planform area). These are calculations that should be done initially. Also, the speed of the plane will depend on the drag induced by the airfoil, since

   thrust = drag in straight and level flight

   The total drag of the airframe will depend largely on the induced drag of the airfoil, but also on the profile drag of the fuselage and other factors.

4 A Critique of the KS theory

While there is much to commend the KS theory there are a number of valid objections or criticisms that can be raised. Among these are:

1. the thrust values calculated seem to be rather low. Is this because EDF is an inefficient way of pwoering a plane, or because the estimates are incorrect ?
2. The power required for flight, as calculated, seems to be on the low side. General observation of models of all types flying in normal conditions seem to suggest that more power should be needed. It is obvious that applying more power will result in faster flight, and it may simply be that the model aircraft fraternity prefers to fly over-powered, rather than under=powered planes, and thus has a high expectation for thrust which is needed.
3. A serious compaint is, however, the following: If in the geometry of the plane it is designed that Ai = Ae (and this happens at least when the EDF fan unit is used without ducts, as is often the case on simple models) then calculation would show that ve = vi so dv= 0. However, in that case it would result that T = 0 and the plane would NOT fly. However, common observation is that such planes DO fly, and often very successfully. This is an important problem. One resolution of this is that the theory ignores (or under rates) power applied at propellor.

5 References

Klaus Scharnhorst, 2005 which is available from SMAC
Propeller thrust calculation http://www.grc.nasa.gov/WWW/k-12/airplane/propth.html
Duct design http://electricjet.blogspot.co.uk/2009/04/electric-ducted-fans-duct-design.html

6 Glossary

Brushless	an alternating current, synchronous, electric motor
duct	the tubes leading air to the EDF and from it
EDF	Electric Ducted Fan
IC	Internal combustion [engine, usually methanol powered, also using nitromethane]